INVESTIGATION OF COMPLEX HEAT TRANSFER AT THE INITIAL SEGMENT OF A TUBE

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On the basis of integral relationships for a boundary layer and on the basis of applying the hypothesis of the radiant-equilibrium layer in the vicinity of a heat-receiving wall, we have derived the theoretical relationships and have proposed a method of calculating the complex transfer of heat for the initial segment of a tube and beyond.

References [1–4 and others] are devoted to the problem of convective heat transfer at the initial segment of a tube. There are very few references [5, 6] dealing with the problem of complex heat transfer. We present the results of a theoretical and experimental investigation of complex heat transfer, conducted on a test stand consisting of three series-connected cooling sections 1 and 2 meters long, each with an inside diameter of 0.15 m ($l/d < 24$).

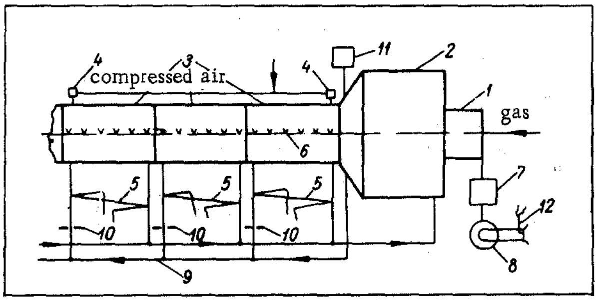

Fig. 1. Experimental apparatus: 1) burner; 2) combustion chamber; 3) sections of a calorimeter (1, 2, 3); 4) thermocouple to measure combustion product temperature; 5) hyperthermocouple to measure cooling water temperature; 6) thermocouples to measure calorimeter wall temperature; 7) electric furnace for heating air with air injection; 8) fan; 9) pipeline for supplying cooling water; 10) disk for measuring water flow rate; 11) gas analyzer; 12) pneumometric pipe with a micromanometer for measuring water flow rate.

The theoretical study of complex heat transfer was conducted on the basis of integral relationships for the boundary layer [7] and the utilization of the hypothesis of a radiant-equilibrium layer close to the wall to which the flow of heat is directed [8].

For the turbulent motion of a fluid in a tube, if we assume a velocity distribution

$$\frac{w}{w_s} = \left(\frac{y}{\delta}\right)^{1/7},$$

and a hydraulic resistance following the Blasius law, the approximate solution for the equation of the conservation of momentum will make it possible to derive a formula for practical calculations of local boundary-layer thickness values in the form of [4]

$$\frac{\delta}{r_0} = \frac{0.15}{\mathrm{Re}^{0.312}}\left(\frac{z}{r_0}\right)^{1.25}. \tag{1}$$

The mean boundary-layer thickness δ_m in a segment of finite tube length (from z_1 to z_2) can be determined by integration of expression (1):

$$\frac{\delta_m}{r_0} = \frac{0.0667\left[\left(\frac{z_2}{r_0}\right)^{2.25} - \left(\frac{z_1}{r_0}\right)^{2.25}\right]}{\mathrm{Re}^{0.312}\left(\frac{z_2}{r_0} - \frac{z_1}{r_0}\right)}. \tag{2}$$

In examining the nonisothermal motion of a gas (Pr = 1) for small pressure gradients, the thicknesses of the hydraulic and thermal layers may be assumed to be equal. Comparison of the equations of motion and of the conservation of energy under these conditions reveals that the velocity and temperature profiles are similar, i.e.,

$$\frac{w}{w_s} = \frac{T - T_w}{T_s - T_w} = \left(\frac{y}{\delta}\right)^{1/7}. \tag{3}$$

For some section z from the equation of the conservation of energy we can derive a relationship for the mean flow temperature T_m in the following form:

$$T_m = \left(0.295\frac{\delta}{r_0}\left(1 - \frac{\delta}{2r_0}\right)(4.56T_s + T_w) + \left(1 - \frac{\delta}{r_0}\right)^2 T_s\right) \times \left(1.64\frac{\delta}{r_0}\left(1 - \frac{\delta}{2r_0}\right) + \left(1 - \frac{\delta}{r_0}\right)^2\right)^{-1}. \tag{4}$$

The total heat flux transmitted from the radiating medium as a result of complex heat transfer can be defined in this case as

$$q = (\alpha_c + \alpha_l)(T_m - T_w) = \alpha_{to}(T_m - T_w). \tag{5}$$

Using the hypothesis of the radiant-equilibrium layer which is separated from the heat-receiving wall through a distance l_s, we can present the expression for the coefficient of radiative heat exchange in the form

$$\alpha_l = \frac{\sigma_0}{\frac{1}{A_w} - \frac{1}{2}}\left[\frac{m^4 T_m^4 - T_w^4}{T_m - T_w}\right], \tag{6}$$

where $m = T_\delta/T_m$.

If we assume that the distance l_s is smaller than the thickness of the laminar sublayer δ_l ($l_s < \delta_l$), and if the temperature of the medium in the sublayer varies in proportion to the distance from the wall, the coefficient m may be expressed as

$$m = \frac{T_\delta}{T_m} = \frac{T_w}{T_m} + \frac{T_l - T_w}{T_m}\,\frac{l_s}{\delta_л}. \qquad (7)$$

The temperature T_l can be determined from (3) under the condition that $y = \delta_l$, when $T = T_l$:

$$\frac{T_l - T_w}{T_s - T_w} = \left(\frac{\delta_l}{\delta}\right)^{1/7}. \qquad (8)$$

Joint consideration of the velocity profile at the boundary of the laminar sublayer $w_l/w_s = (\delta_l/\delta)^{1/7}$ and

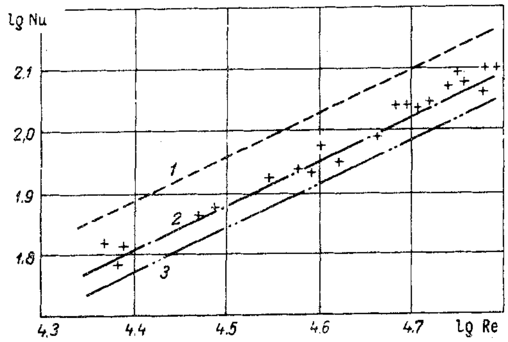

Fig. 2. Functions $Nu = \psi(Re, Pr)$ for separate sections. The experimental points pertain to the entire experimental segment $l/d = 24$: 1) section 1, $Nu = 0.0252\ Re^{0.8}\ Pr^{0.4}$; 2) section 2, $Nu = 0.0207\ Re^{0.8}\ Pr^{0.4}$; 3) section 3, $Nu = 0.0195\ Re^{0.8}\ Pr^{0.4}$.

of the expression for velocity $w_l = 0.0228\rho(w_s^2/\mu) \times (\nu/w_s\delta)^{0.25}\,\delta_l$ (from the relationship for the force of friction at the wall [7]) we can derive a relationship defining the thickness of the laminar sublayer

$$\frac{\delta_l}{\delta} = \frac{82.5}{\left(\dfrac{w_s\delta}{\nu}\right)^{7/8}}. \qquad (9)$$

With consideration of (8) and (9) expression (7) will make it possible to derive a relationship for the determination of l_s,

$$\frac{d}{l_s} = 0{,}0456\,\frac{T_s - T_w}{mT_m - T_w}\left(\frac{w_s\delta}{\nu}\right)^{3/4}\frac{1}{\delta/r_0}. \qquad (10)$$

The convective heat-transfer coefficient is defined by means of the expression

$$\alpha_c = \frac{\lambda}{l_s}\,\frac{mT_m - T_w}{T_m - T_w}. \qquad (11)$$

From (6) and (11) the total heat-transfer coefficient

$$\alpha_{to} = \frac{\sigma_0(m^4T_m^4 - T_w^4)}{\left(\dfrac{1}{A_w} - \dfrac{1}{2}\right)(T_m - T_w)} + \frac{\lambda}{l_s}\,\frac{(mT_m - T_w)}{(T_m - T_w)}. \qquad (12)$$

The study conducted on the experimental installation (Fig. 1) made it possible to derive theoretical recommendations for the determination of the total heat-

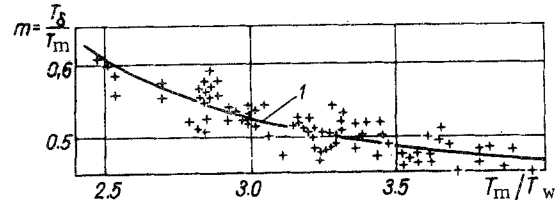

Fig. 3. Dependence for $m = \varphi(T_m/T_w)$.

$$1 - m = 0.44 + \frac{6.6}{(T_m/T_w)^4}$$

transfer coefficient. The processing was accomplished with experimental data for which the divergence in the balance with respect to water and gas did not exceed 5%. The convective heat-transfer coefficient α_c was defined (Fig. 2) from data pertaining to the passage of heated air through the test stand.

In this study of complex heat transfer the products of gas combustion were let into the stand, the temperature of these products varying from 700 to 1250° C, with the Re numbers varying from 12 000 to 35 000.

The total heat-transfer coefficient α_{to} was determined from experimental data on the transfer of heat between the products of combustion and the heat-receiving wall, according to the relationship

$$\alpha_{to} = \frac{Q}{\pi\,dl\,(T_m - T_w)}. \qquad (13)$$

The coefficient α_l of radiative heat exchange was defined as the difference between the total heat-transfer coefficient α_{to} and the coefficient α_c of convective heat transfer.

The distance l_s from the heat-receiving wall to the radiant-equilibrium layer was determined in accordance with (10). The temperature T_s beyond the limits of the boundary layer in this case was determined

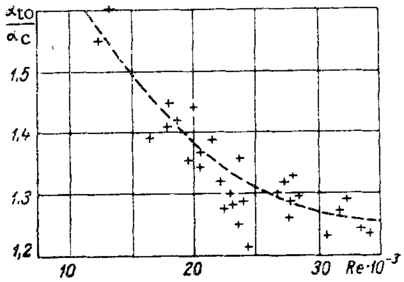

Fig. 4. Function for $\alpha_{to}/\alpha_c = \psi(Re)$ constructed for the second section of the calorimeter.

from (4), while for a certain value of α_l the coefficient m was determined from (6). In formulas (10) and (4) δ was assumed to be the mean thickness of the boundary layer within the limits of each section, calculated from (2).

Processing of the experimental data demonstrated that the coefficient m for the structure of the temperature layer is independent of the Re number. With a change in the temperature of the medium within a wide range ($T_m/T_w = 2.5-5$) we derived a relationship for m (Fig. 3) which is in satisfactory agreement with the experimental data of other authors [5]:

$$m = 0.44 + \frac{6.6}{(T_m/T_w)^4}. \qquad (14)$$

Figure 4 shows the experimental data in the form of the relationship $\alpha_{to}/\alpha_c = \psi(\mathrm{Re})$. With varying Re numbers no constancy is observed in the ratio α_{to}/α_c, but rather a trend toward a reduction in this ratio, explained by the great variation in the convective components relative to the radiative component as the hydrodynamics of the flow is changed.

The position of the radiant-equilibrium layer l_s is determined by the hydrodynamics, composition, temperature of the medium, as well as by the length of the initial segment.

The effect of the hydrodynamics is a predominant factor. With increasing Re the distance l_s from the wall to the radiant-equilibrium layer diminishes. With constant Re there is an increase in l_s in proportion to the length of the initial segment of the tube.

To simplify the calculation Eq. (10) can be approximated on the basis of our experimental data by the simpler expression

$$\frac{d}{l_s} = 0.112 \frac{\mathrm{Re}^{0.829}}{\left(\frac{l}{d}\right)^{0.2}}. \qquad (15)$$

An evaluation of l_s has demonstrated that it is smaller than the thickness of the laminar sublayer.

Thus the adopted assumption that $l_s < \delta_l$ reflects the physical pattern of heat transfer near the wall.

Comparison of α_c and α_{to}, derived from (11) and (12), with the experimental data has demonstrated that the divergence between these does not exceed 10%.

The calculation of complex heat transfer in turbulent motion of a radiating medium at the initial segment of a tube can be accomplished in the following sequence.

For a certain mean flow temperature T_m and for a certain wall temperature T_w we can determine the value of the coefficient m from (14). The values of the local and mean coefficient α_l of radiative heat exchange are calculated from (6).

Using (1) or (2), (4), (10), and (11), we determine the value of the local or mean coefficient of convective heat transfer. We find the magnitude of the total specific flow from (5). Verification of the validity of the calculation can be accomplished with respect to T_w.

If we assume that $\delta/r_0 = 1$, the proposed method may be used to calculate complex heat transfer beyond the limits of the initial segment.

NOTATION

Here l is the length of the initial section; d and r_0 are the diameter and radius of the tube; z is the coordinate in the direction of medium motion; y is the coordinate over the tube section; δ and δ_m are local and mean thicknesses of the hydrodynamic boundary layer; δ_l is the thickness of the laminar sublayer; w is the velocity within the boundary layer; w_s is the velocity in the undisturbed part of the flow; Re is the Reynolds number; T is the mean temperature within the boundary layer; T_s is the medium temperature in the undisturbed part of the flow; T_w is the tube wall temperature; T_m is the mean temperature of medium flow; α_c is the convective heat transfer coefficient; α_l is the radiant heat transfer coefficient; α_{to} is the total heat transfer coefficient; l_s is the distance from a wall to a radiant-equilibrium layer; σ_0 is the Boltzmann constant; A_w is the absorptivity of a wall; T_δ is the mean temperature of the radiant equilibrium layer; $m = T_\delta/T_m$ is the ratio of the radiant-equilibrium layer temperature T_δ to the mean medium temperature T_m; T_l is the temperature at the boundary between the laminar sublayer and the turbulent core of the medium, ν is the kinematic viscosity; λ is the thermal conductivity of the medium at the mean temperature of the boundary layer; Q is the total quantity of heat in separate sections of the initial segment.

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